Developing the tools for “boosted frame” calculations.

J.-L. Vay*[1,4]

in collaboration with

W.M. Fawley[1], A. Friedman[2,4], M.A. Furman[1] ,
C.G. Geddes*[1], D.P. Grote[2,4], S. Markidis[1,3,4]

[1]Lawrence Berkeley National Laboratory, CA
[2]Lawrence Livermore National Laboratory, CA
[3]University of Illinois, Urbana-Champaign, IL
[4]Heavy Ion Fusion Science Virtual National Laboratory

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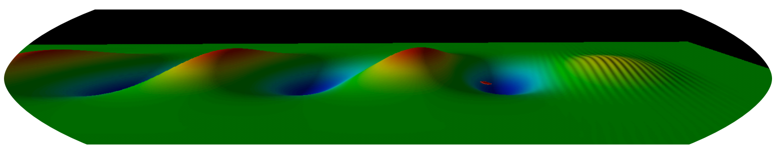
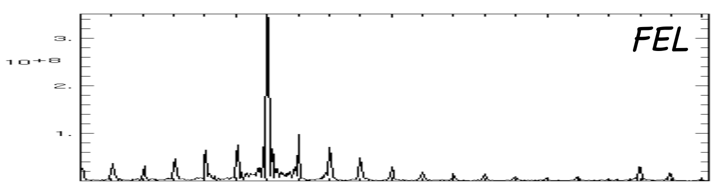
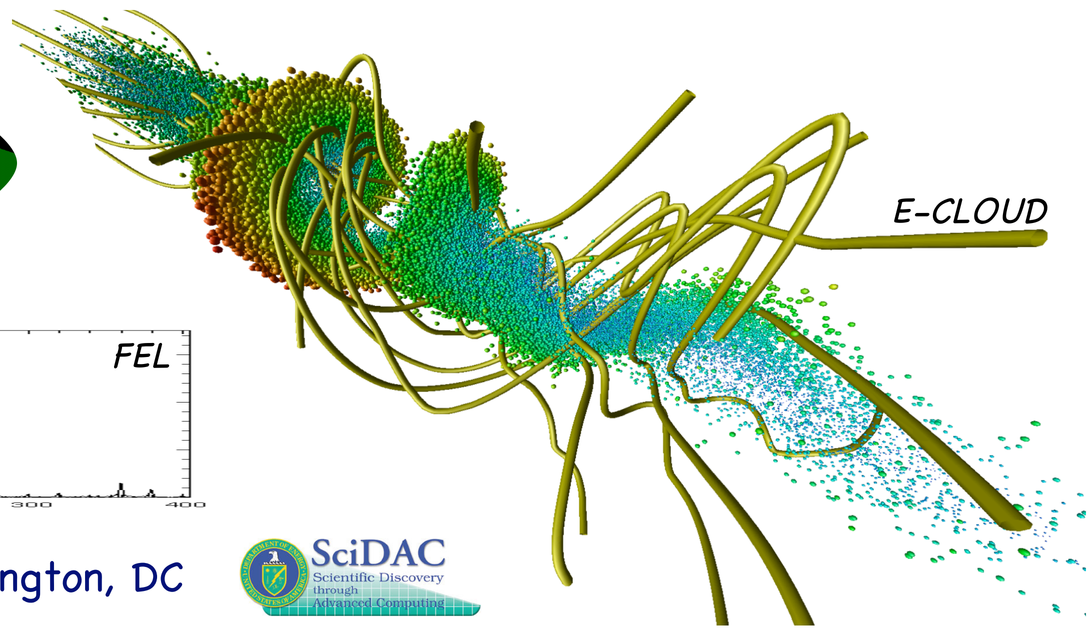


LWFA





E-CLOUD

SCIDAC Review, April 21-22, 2009, Washington, DC

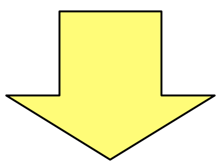
Concept

- # of computational steps grows with the full range of space and time scales involved

- key observation
  - ***range*** of space and time scales is ***not*** a Lorentz invariant*

    scales as γ^2 in x and t

  - the optimum frame to minimize the range is not *necessarily* the lab frame



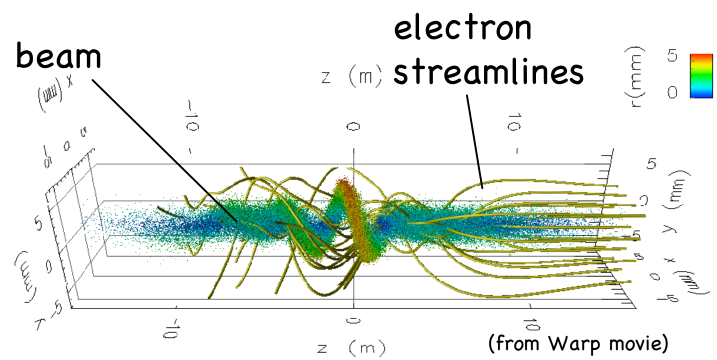
Choosing optimum frame of reference to minimize range can lead to dramatic speed-up for relativistic matter-matter or light-matter interactions.

*J.-L. Vay, *Phys. Rev. Lett.* **98**, 130405 (2007)

Calculation of e-cloud induced TMC instability of a proton bunch

- Proton energy: γ=500 in Lab
- L= 5 km, continuous focusing

Code: Warp (Particle-In-Cell)



CPU time (2 quad-core procs):
- lab frame: >2 weeks
- frame with γ^2=512: <30 min

Speedup x1000

Seems simple but ⚠. Algorithms which work in one frame may break in another. Example: the Boris particle pusher.

- Boris pusher ubiquitous
  - In first attempt of e-cloud calculation using the Boris pusher, the beam was lost in a few betatron periods!
  - Position push: $\mathbf{X}^{n+1/2} = \mathbf{X}^{n-1/2} + \mathbf{V}^n \Delta t$ -- no issue
  - Velocity push: $\gamma^{n+1}\mathbf{V}^{n+1} = \gamma^n\mathbf{V}^n + \frac{q\Delta t}{m}\left(\mathbf{E}^{n+1/2} + \frac{\gamma^{n+1}\mathbf{V}^{n+1} + \gamma^n\mathbf{V}^n}{2\,\gamma^{n+1/2}} \times \mathbf{B}^{n+1/2}\right)$

    issue: $\mathbf{E}+v\times\mathbf{B}=0$ implies $\mathbf{E}=\mathbf{B}=0$ => large errors when $\mathbf{E}+v\times\mathbf{B}\approx 0$ (e.g. relativistic beams).

- Solution
  - Velocity push: $\gamma^{n+1}\mathbf{V}^{n+1} = \gamma^n\mathbf{V}^n + \frac{q\Delta t}{m}\left(\mathbf{E}^{n+1/2} + \frac{\mathbf{V}^{n+1} + \mathbf{V}^n}{2} \times \mathbf{B}^{n+1/2}\right)$

- Not used before because of implicitness. We solved it analytically*

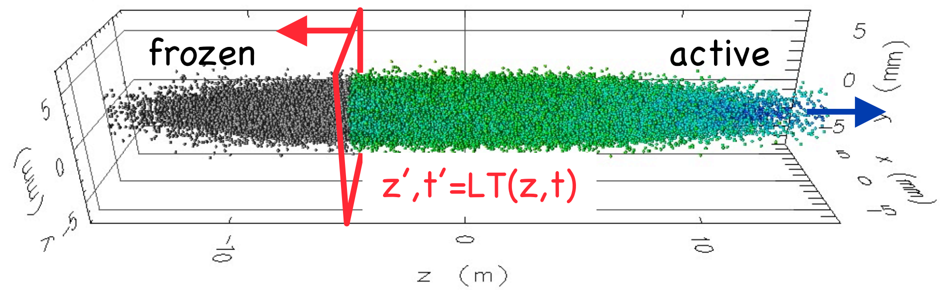
$$\begin{cases} \gamma^{i+1} = \sqrt{\dfrac{\sigma + \sqrt{\sigma^2 + 4(\tau^2 + u^{*2})}}{2}} \\ \mathbf{u}^{i+1} = [\mathbf{u}' + (\mathbf{u}'\cdot\mathbf{t})\mathbf{t} + \mathbf{u}'\times\mathbf{t}]/(1+t^2) \end{cases}$$

(with $\mathbf{u} = \gamma\mathbf{v}$, $\mathbf{u}' = \mathbf{u}^i + \frac{q\Delta t}{m}\left(\mathbf{E}^{i+1/2} + \frac{\mathbf{v}^i}{2}\times\mathbf{B}^{i+1/2}\right)$, $\boldsymbol{\tau} = (q\Delta t/2m)\mathbf{B}^{i+1/2}$; $u^* = \mathbf{u}'\cdot\boldsymbol{\tau}/c$, $\sigma = \gamma'^2 - \tau^2$, $\gamma' = \sqrt{1+u'^2/c^2}$, $\mathbf{t} = \boldsymbol{\tau}/\gamma^{i+1}$).

*J.-L. Vay, *Phys. Plasmas* **15**, 056701 (2008)

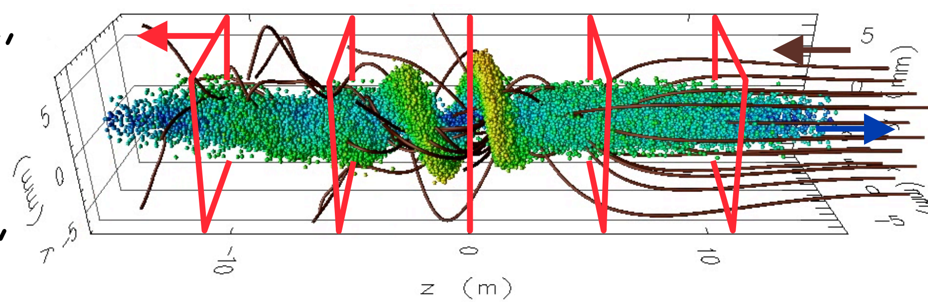
Other complication: input/output

- Often, initial conditions known and output desired in laboratory frame
  - relativity of simultaneity => inject/collect at plane(s) ⊥ to direction of boost.
- Injection through a moving plane in boosted frame (fix in lab frame)
  - fields include frozen particles,
  - same for laser in EM calculations.



- Diagnostics: collect data at a collection of planes
  - fixed in lab fr., moving in boosted fr.,
  - interpolation in space and/or time,
  - already done routinely with Warp

  for comparison with experimental data, often known at given stations in lab.

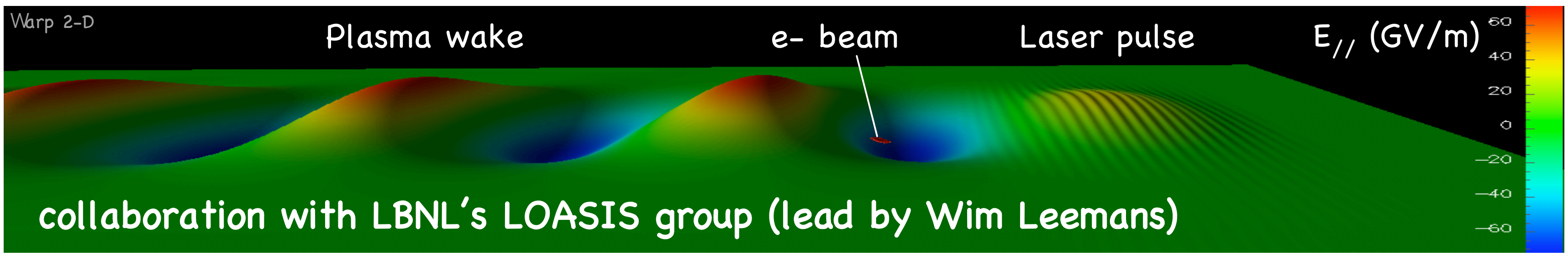


Application to Laser-plasma wakefield accelerators

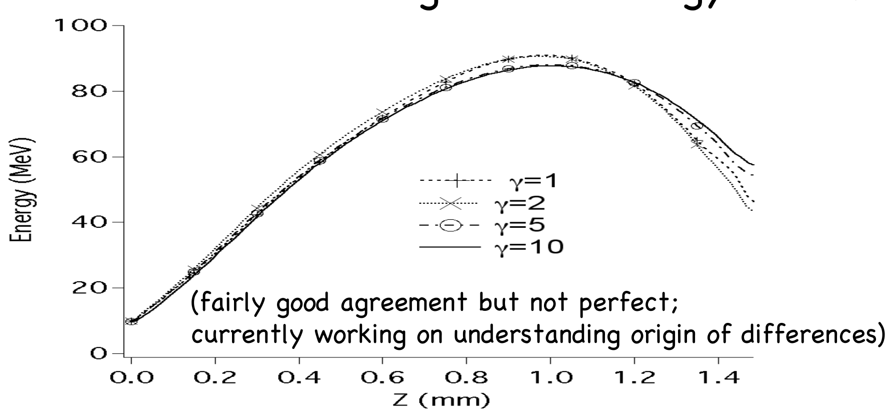
- New electromagnetic solver implemented in Warp (SBIR funding)
  - scaling test (3-D decomp)

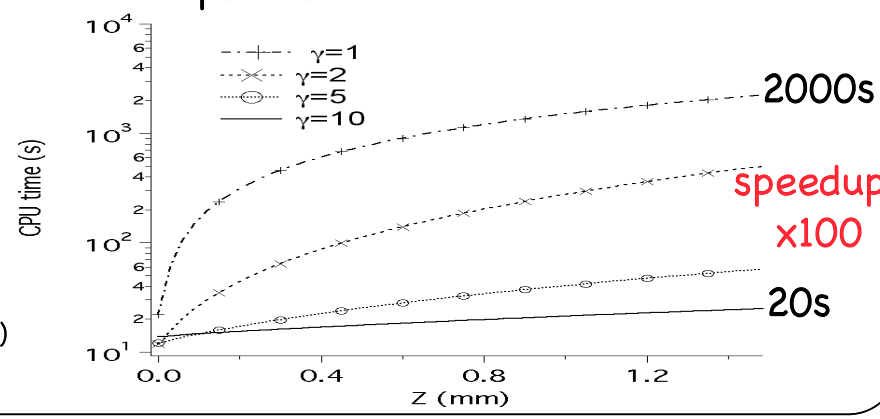
# procs	256 (8x8x4)	512 (8x8x8)	1024 (8x8x16)
# cell, particles	$1{,}024^2$x512, 100M	$1{,}024^3$, 200M	$1{,}024^2$x2,048, 400M
Time ratio	1.	1.04	1.12

- Applied to modeling of one stage of LWFA (2-D for now, 3-D to follow)



Average beam energy and CPU time vs position in lab frame





Other accomplishements; future work

- Accelerator lattice in Warp: added linear maps, boosted frame tracking
  - will apply to e-cloud simulations for SPS, LHC, ILC, etc.

- W. Fawley (LDRD LBNL) applying Warp to numerical study of Free Electron Lasers (FEL) and Coherent Synchrotron Radiation (CSR)
  - detailed benchmarking of FEL physics: spontaneous emission, coherent spontaneous emission, amplifier gain, sideband emission effects of subharmonic bunching, etc.,
  - simulation of CSR: examine transverse size effects normally neglected by theory and computationally prohibitively expensive under normal lab frame E&M calculations.

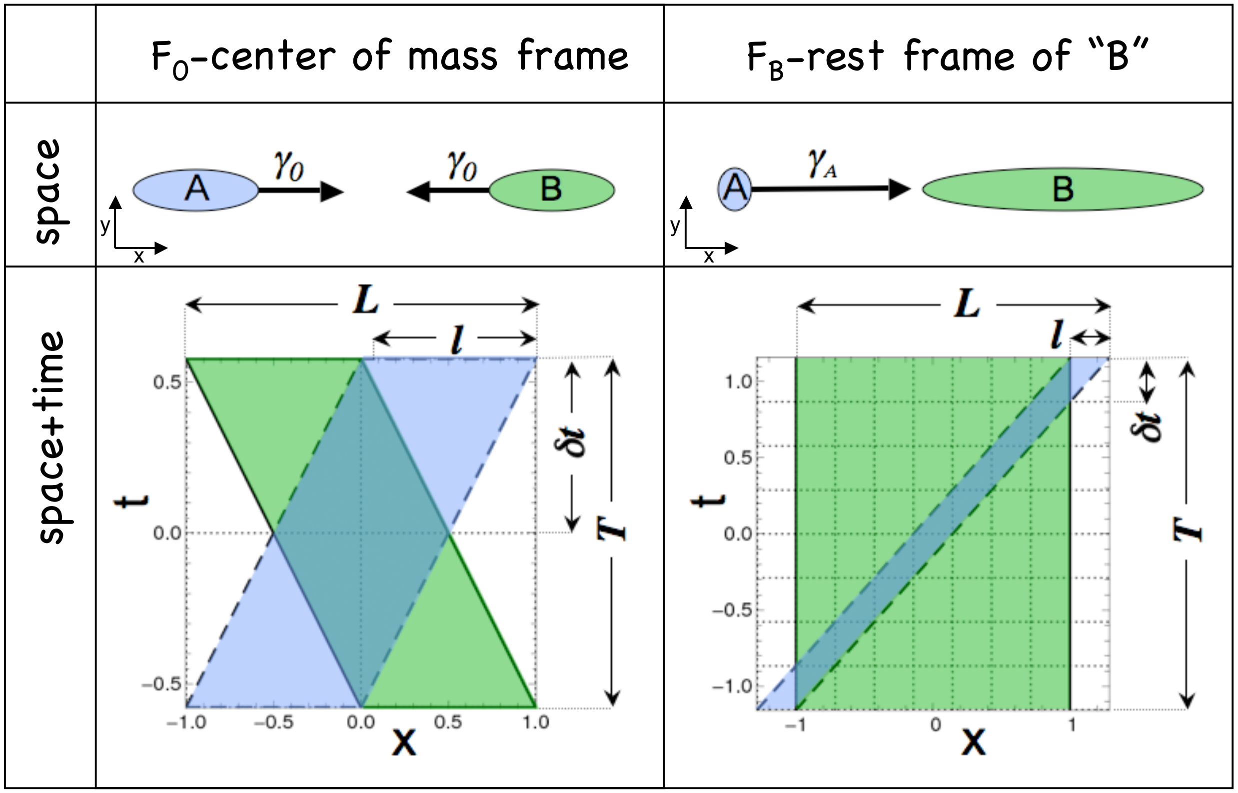
- Pursue development and detailed algorithmic/physics studies of boosted frame calc. for problems of interest to HEP: LWFA, E-cloud, FEL, CSR, …

- Apply Warp's novel EM solver with mesh refinement (MR) in lab frame and boosted frame simulations
  - LWFA stage in 3-D: required resolution may vary by more than 2 orders of magnitude in transverse directions. Applying MR:
    - up-to 10^4 saving on # grid cells for 10 GeV,
    - up-to 10^8 saving on # grid cells for 1 TeV.

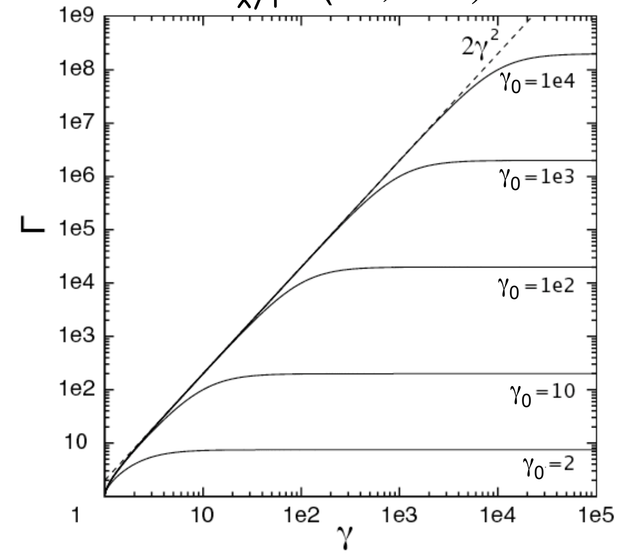
BACKUPS

Range of space and time scale of a simple system
two identical objects crossing each other

same event as seen in two frames



$\Gamma_{x/t} = (L/l,\ T/\delta t)$*



$$\Gamma_{x/t} \propto \gamma^2$$

$$\Gamma = \Gamma_x \cdot \Gamma_t \propto \gamma^4$$

- Γ is not invariant under the Lorentz transformation.
- There exists an optimum frame which minimizes ranges.
- For PIC, Vlasov, fluid methods, cost$\propto\Gamma \Rightarrow$ huge penalty if calculation not performed in optimum frame!

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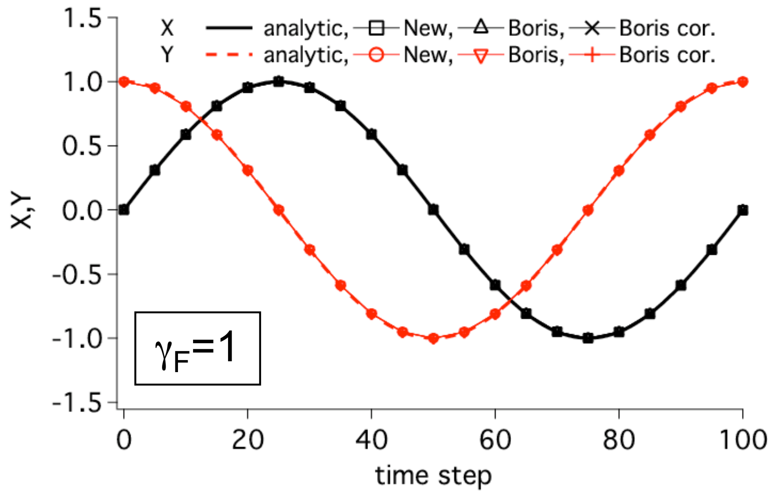
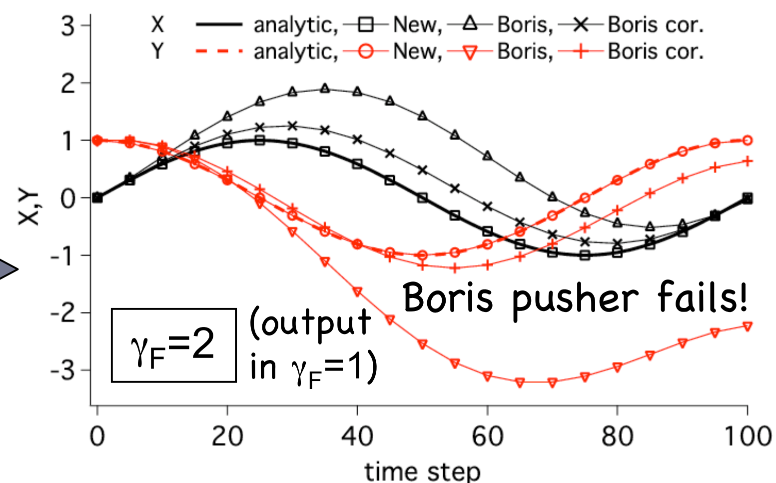
- New pusher*
  - Velocity push: $\gamma^{n+1}\mathbf{V}^{n+1} = \gamma^n\mathbf{V}^n + \frac{q\Delta t}{m}\left(\mathbf{E}^{n+1/2} + \frac{\mathbf{V}^{n+1} + \mathbf{V}^n}{2} \times \mathbf{B}^{n+1/2}\right)$
- Test one particle in constant **B**

*J.-L. Vay, *Phys. Plasmas* **15**, 056701 (2008)



becomes ExB drift

in boosted frame



Lorentz boosted simulations applied to various problems

3-D electron driven TMC instability (Warp-LBNL), **x1000**

2-D free electron laser toy problem (Warp-LBNL), **x45,000*/****

3-D coherent synchrotron emission (Warp-LBNL), **x350***

2-D laser-plasma acceleration (Warp-LBNL), **x100***

1-D laser-plasma acceleration (Vorpal-Tech-X), **x1,500**

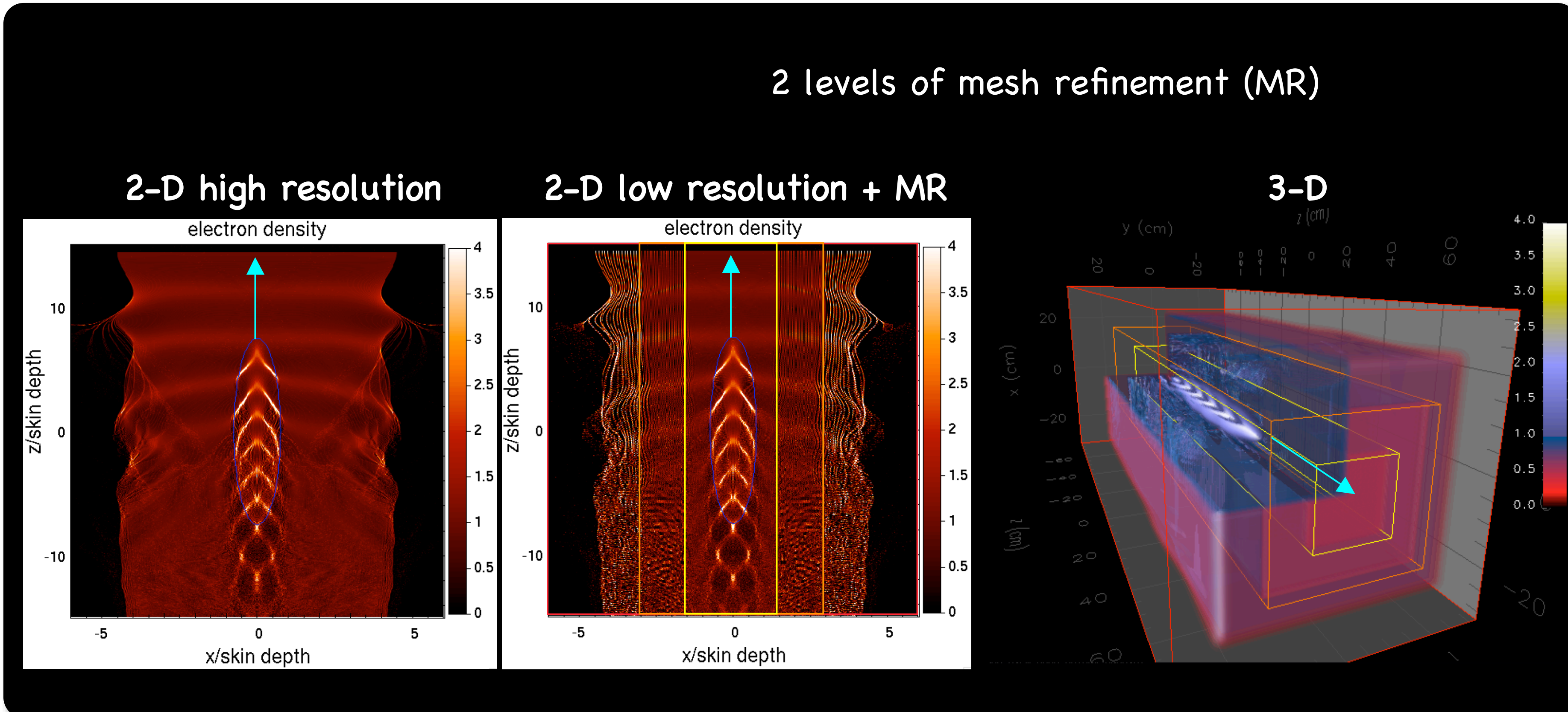
laser-plasma acceleration (Osiris-IST, Portugal) **x150** 2-D, **x75** 3-D

*estimated

**compared to PIC simulation in lab frame. PIC in boosted frame slower than Eikonal codes but allows study of matching ramp and sub-harmonic bunching which are not accessible to Eikonal codes.

Other applications: astrophysics,...?

Electromagnetic MR simulation of beam-induced plasma wake with Warp



There simulations used the same time steps for all refinement levels.

The implementation of separate time steps for each refinement level is underway.